The Embroidery Problem

Estie George Steve Joe

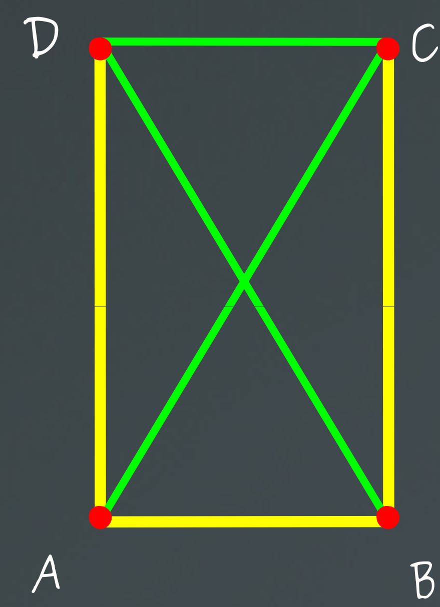
Irina Girish Joondong

Stony Brook University, NY

Rural Postman Problem

Given : Edge-Weighted Graph
$G(V,E)$ & $E' \subseteq E$

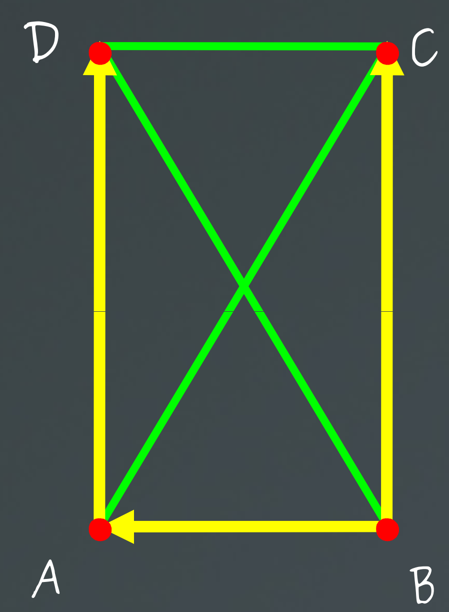
Find : Min-Weight Closed
Walk traversing E' at least
once



Stacker Crane Problem

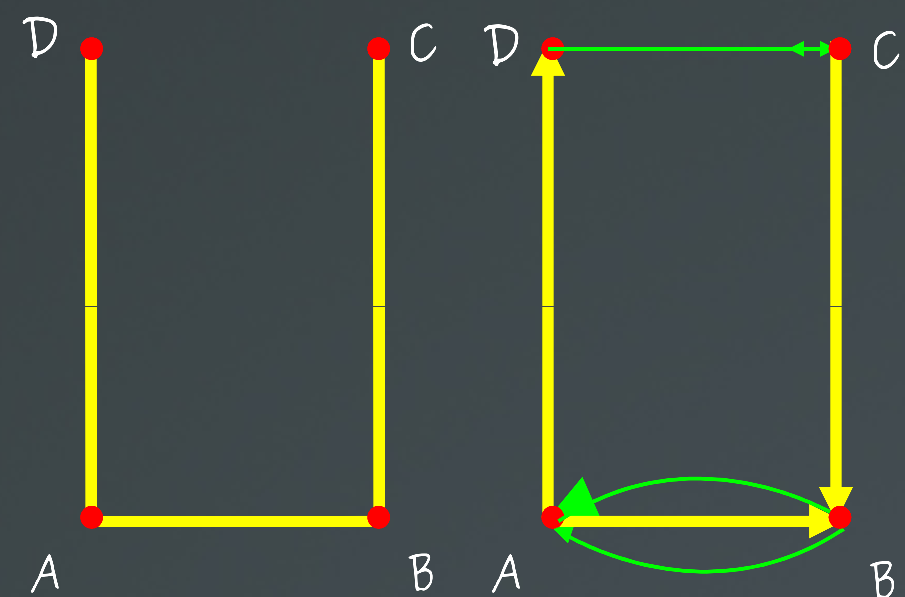
Given : Edge Weighted Graph
$G(V,E)$ & $E' \subseteq E$ and E' is
directed

Find : Min Weight Closed Walk
traversing directed E'
at least once



Embroidery Problem

Now let's try to Embroider
this design on a Cloth



Embroidery Problem

Model

Euclidean embedding of design given in R^2

Single continuous piece of thread

Front design (E') Exact (no repetitions)

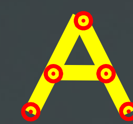
Back edges come from VxV (may repeat)

End at Starting Point (possibly to tie the thread)

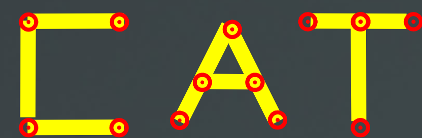
Embroidery Problem

Different Cases

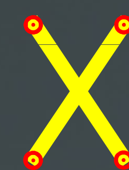
Entire Design One Connected Component



Multiple Components



Disjoint Segments (possibly crossing)



Objective Functions

Minimize Total Length of thread

Minimize Back Length (wasted thread)

Related Work

- Rural Postman : 3/2 apx

  H. A. Eiselt, M. Gendreau, and G. Laporte, 1995

- Stacker Crane : 9/5 apx

  G. N. Frederickson, M. S. Hecht, and C. E. Kim, 1978

- Cross-Stitching (special kind of embroidery)

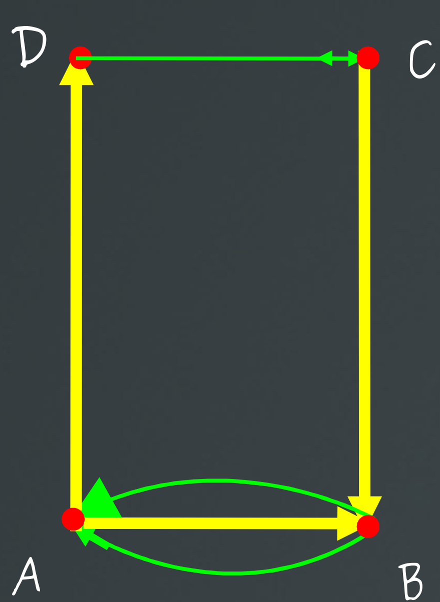
  T. Biedl, J. D. Horton, and A. Lopez-Ortiz, CCCG'05

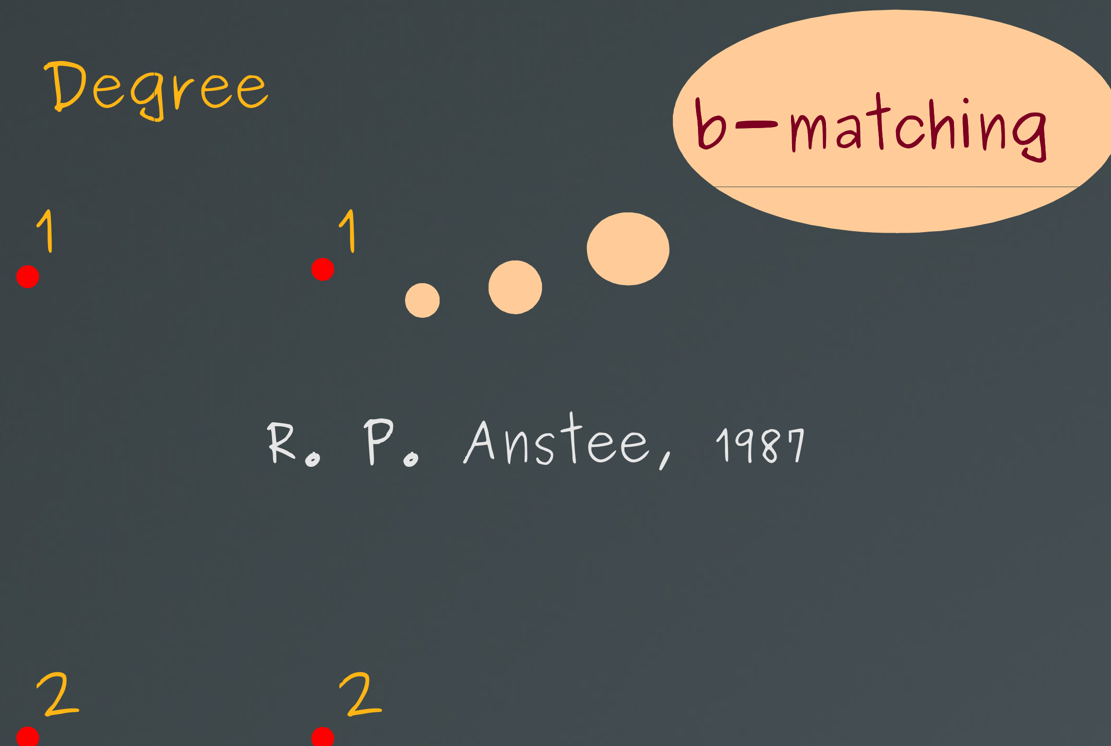
One Connected Component

(critical) Observation :

T is an Embroidery tour, iff at any node

Front Degree = Back Degree





R. P. Anstee, 1987

Multiple Connected Components

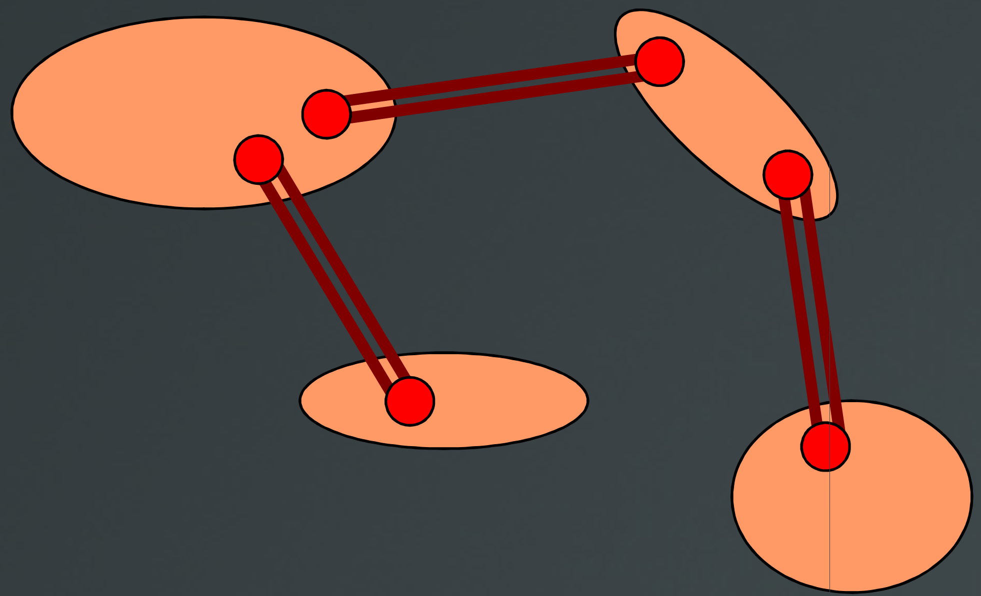
Apply b−matching

Insufficient !! The components after b−matching may be disconnected

In fact NP − Complete, simple reduction from Euclidean TSP

Approximation Algorithm

Start with b-matching



MST between Connected Components

(doubled)

Analysis

Min-Total Length

b-matching <= Front

MST <= Back

T_{apx} = Front + b-matching + 2*MST

<= 2*(Front + Back) = 2*OPT

T_{apx} <= 2*OPT

Min-Back Length

b-matching <= Back

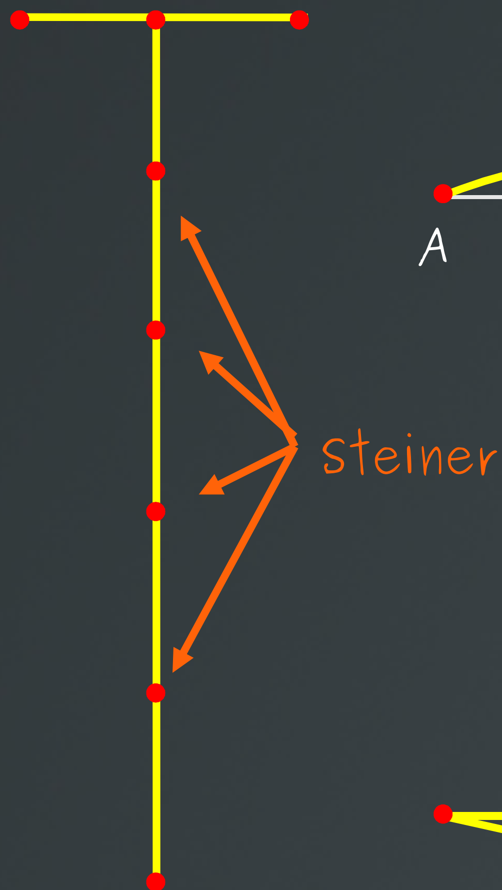
MST <= Back

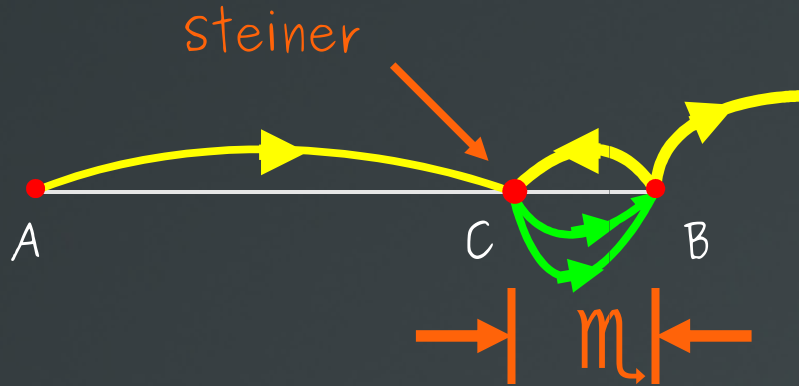
B_{apx} = b-matching + 2*MST

<= Back + 2*Back = 3*OPT_B

B_{apx} <= 3*OPT

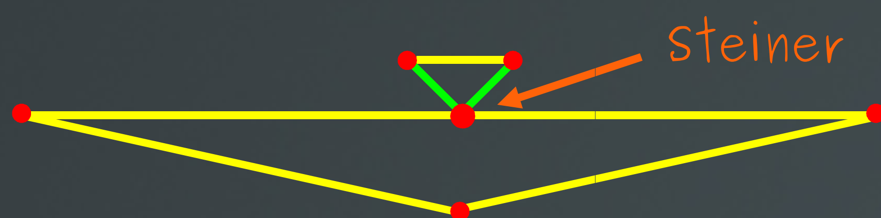
Adding Steiner Points (along edges)





Cheating

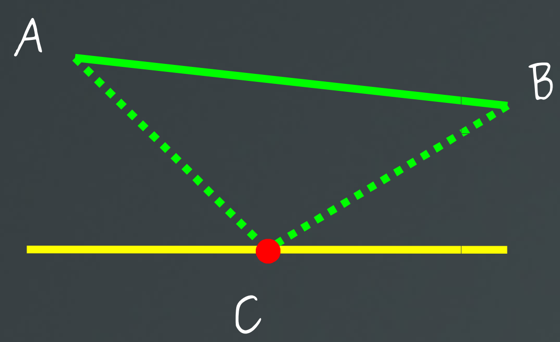
Zero Back Length



Multiple Components

One Connected Component (Steiner Case)

Lemma 1 : There exists T_{opt} with no Steiner Points on Edges, except at ends (cheating ones)



Lemma 2 : There exists T_{opt} with no two Back Edges incident to common vertex

With Steiner Points

One Connected Component

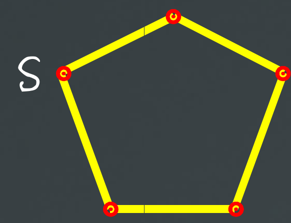
- Min Weight Perfect Matching between odd degree vertices ! (instead of b-matching)

Multiple Components (approximation)

- Use Perfect matching (odd ones) instead of b-matching and MST between components (same 2-Apx and 3-Apx as one without using Steiner points)

Do Steiner Points Help ?

Yes, Of course !



Zero Back Length

But Not Much if we consider Total Length

Theorem : $OPT_{steiner} >= ½ * OPT_{non-steiner}$

Conclusions

- Cute Problem
- Exact Polynomial Algorithms for one connected component case
- Hardness and Approximation results for multiple component case
- PTAS for disjoint segment case

Open Problems

More practical variants of the problem

Improve approximation bounds to something
christofides-like 3/2 ?   PTAS ??

Any other applications ?

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Bonnie Skiena for posing the question

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Many Thanks !